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Solution of Linear Stochastic Differential Equations for the Impact of Inflation and Long-Term Growth Trends on Bank Valuation

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Abstract. *This study investigates the dynamics of Nigerian bank stock prices using an exponential-growth geometric Brownian motion framework. The model extends the classical GBM by incorporating a growth-adjustment parameter k in the drift term, allowing it to capture both deterministic long-term growth and stochastic fluctuations. Three Brownian-motion paths are first presented to illustrate the underlying randomness, followed by sample price paths demonstrating how the same growth rate can produce both upward and downward trajectories depending on the shock realizations. A sensitivity analysis is then conducted to assess the effect of k on the simulated paths. The results show that higher values of k amplify exponential growth or decay and increase dispersion in absolute valuation. This framework provides insight into how long-term growth expectations interact with short-term volatility in the Nigerian banking sector.*

Keywords: stochastic fluctuations; bank valuation; geometric Brownian motion; inflation; long-term growth; stock prices.

1. Introduction

Bank valuation in Nigeria takes place in a changing macroeconomic environment. Inflation can erode real earnings and purchasing power, while economic expansion and financial deepening can support long-term nominal growth. Conventional GBM uses a constant drift without explicitly separating these forces. We define a net growth adjustment $k = g - \pi$, where g is long-term growth and π is inflationary drag, and add k to the baseline drift μ.

Stochastic models have been widely applied to asset prices, investment decisions, insurance risk, and market stability. Osu et al. examined Markov models for Nigerian share-price movements [1]; Amadi et al. studied trend functions with stochastic terms [2] and a matrix approach to stock-price prediction [3]. Optimal investment and ruin-minimization problems were treated by Browne [4]; related reinsurance-investment problems appear in Bai and Guo [5], Liang and Bayraktar [6], and Ihedioha and Osu [7]. Extensions involving jump diffusion, stochastic volatility, and constant-elasticity-of-variance dynamics were investigated by Zhao et al. [8], Gu et al. [9], Lin and Li [10], and Liang et al. [18].

Within the stock-market literature, Osu and Okoroafor quantified random stock-price behavior [11]. Stability and stochastic price dynamics were studied by Davies et al. [12], Osu et al. [13], and Adeosun et al. [14]. Ofomata et al. developed a stochastic stock-price forecasting model [15]; Osu [16] and Ugbebor et al. [17] also examined capital-market price changes. Standard stochastic-calculus arguments underlying these models can be applied alongside related work on random-parameter populations and stochastic differential equations [19, 20, 21].

Existing studies motivate stochastic modeling, but a growth-adjusted GBM makes macroeconomic assumptions explicit and analytically tractable. The objective is to derive the model, establish existence and uniqueness, simulate representative paths, and characterize sensitivity to k. Because no observed bank-price series is fitted, the analysis is theoretical and illustrative rather than an empirical forecast.

2. Materials and Methods

2.1. Standard geometric Brownian motion

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions, and let $\{W_t\}_{t\geq 0}$ be a standard Brownian motion. The standard GBM satisfies

$$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t, \qquad S_0 > 0, \tag{1}$$

where $\mu \in \mathbb{R}$ is the drift and $\sigma \geq 0$ is the volatility.

Theorem 2.1 (Itô's formula). *If X_t satisfies $dX_t = a(t, X_t)\,dt + b(t, X_t)\,dW_t$ and $f \in C^{1,2}$, then*

$$df(t, X_t) = \left(f_t + af_x + \frac{1}{2}b^2 f_{xx}\right) dt + bf_x\,dW_t. \tag{2}$$

Applying Itô's formula to $f(S) = \ln S$ in Equation (1) gives

$$d\ln S_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma\,dW_t, \tag{3}$$

and hence

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right]. \tag{4}$$

2.2. Inflation- and growth-adjusted GBM

Let $k = g - \pi \in \mathbb{R}$ denote the net long-term adjustment. The corrected, well-posed model is

$$dS_t = (\mu + k)S_t\,dt + \sigma S_t\,dW_t, \qquad S_0 = S_{\text{base}} > 0. \tag{5}$$

This replaces the manuscript's time-dependent "initial condition" $S_{\text{base}}e^{kt}$, which cannot be an initial value. The equivalent exponential trend is incorporated in the drift, giving deterministic baseline S_0e^{kt}.

The formulation uses the following assumptions:

1. $S_0 > 0$;
2. $\mu, k \in \mathbb{R}$ are finite constants;
3. $\sigma \geq 0$ is a finite constant; and
4. W_t is a standard Brownian motion adapted to $\{\mathcal{F}_t\}$.

Theorem 2.2 (Existence, uniqueness, and explicit solution)**.** *Under the stated assumptions, Equation* (5) *has a unique strong solution,*

$$S_t = S_0 \exp\left[\left(\mu + k - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right]. \tag{6}$$

Moreover, $S_t > 0$ almost surely for every finite $t \geq 0$.

Proof. The drift $b(s) = (\mu + k)s$ and diffusion coefficient $a(s) = \sigma s$ are globally Lipschitz and satisfy a linear-growth condition; therefore, a unique strong solution exists. For the explicit form, apply Itô's formula to $Y_t = \ln S_t$:

$$dY_t = \left(\mu + k - \frac{1}{2}\sigma^2\right) dt + \sigma\, dW_t. \tag{7}$$

Integrating from 0 to t and exponentiating yields Equation (6). Positivity follows immediately from the exponential representation. $\square$

An equivalent derivation uses $Z_t = e^{-kt}S_t$. The product rule gives

$$dZ_t = \mu Z_t\, dt + \sigma Z_t\, dW_t, \tag{8}$$

so Z_t is a standard GBM and $S_t = e^{kt}Z_t$, which again produces Equation (6).

2.3. Distributional properties and stress metrics

From Equation (6),

$$\ln S_t \sim \mathcal{N}\left(\ln S_0 + \left(\mu + k - \frac{\sigma^2}{2}\right) t,\ \sigma^2 t\right). \tag{9}$$

Consequently,

$$\mathbb{E}[S_t] = S_0 e^{(\mu+k)t}, \tag{10}$$

$$\operatorname{Var}(S_t) = S_0^2 e^{2(\mu+k)t}\left(e^{\sigma^2 t} - 1\right). \tag{11}$$

When $\mu + k < 0$, the expected price declines exponentially and its half-life is

$$t_{1/2} = \frac{\ln 2}{-(\mu + k)}. \tag{12}$$

This quantity provides a direct, interpretable stress metric. The expected price is constant when $\mu + k = 0$ and grows when $\mu + k > 0$.

2.4. Simulation procedure

For a step size Δt, exact discrete simulation of Equation (5) is

$$S_{t+\Delta t} = S_t \exp\left[\left(\mu + k - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\, Z_t\right], \tag{13}$$

where the Z_t are independent standard normal random variables. Exact lognormal stepping avoids the negative simulated prices that can occur under a coarse Euler–Maruyama discretization.

3. Results and Discussion

3.1. Brownian-motion paths

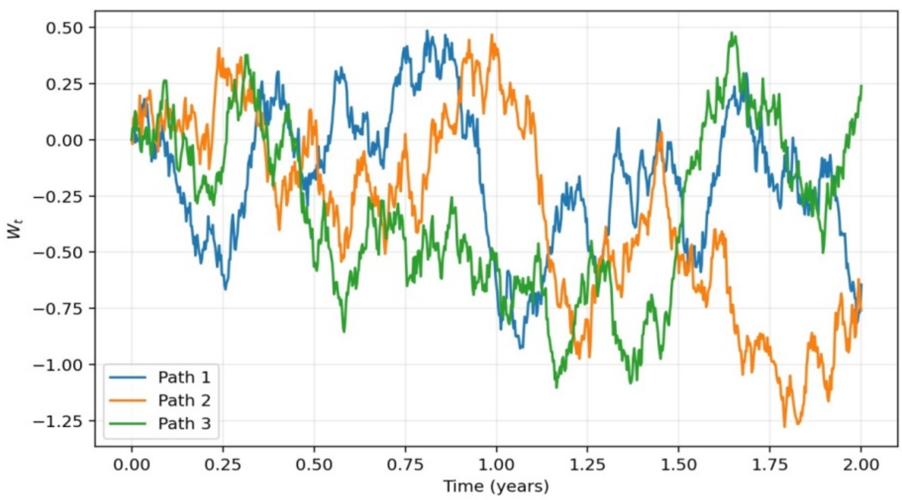

Figure 1: Three sample Brownian-motion paths used to illustrate the stochastic driver.

Figure 1 presents three sample paths of the driving Brownian motion. All paths start at the same point, but their subsequent trajectories diverge. This illustrates how identical initial conditions can produce materially different outcomes once random increments enter the model. These paths provide the stochastic input in Equation (6).

3.2. Growth-adjusted stock-price paths

Figure 2 displays illustrative price paths from the growth-adjusted SDE. The paths may move together initially and then separate because each realization receives different Brownian increments. A positive k raises the common expected growth rate, but adverse shocks may still produce a declining individual path over a finite horizon.

3.3. Sensitivity to the growth-adjustment parameter

Equation (10) establishes three regimes. If $\mu + k > 0$, the expected value grows; if $\mu + k = 0$, it remains constant; and if $\mu + k < 0$, it declines exponentially. Figure 3 illustrates how changing k shifts the simulated level. For fixed σ, k changes the mean

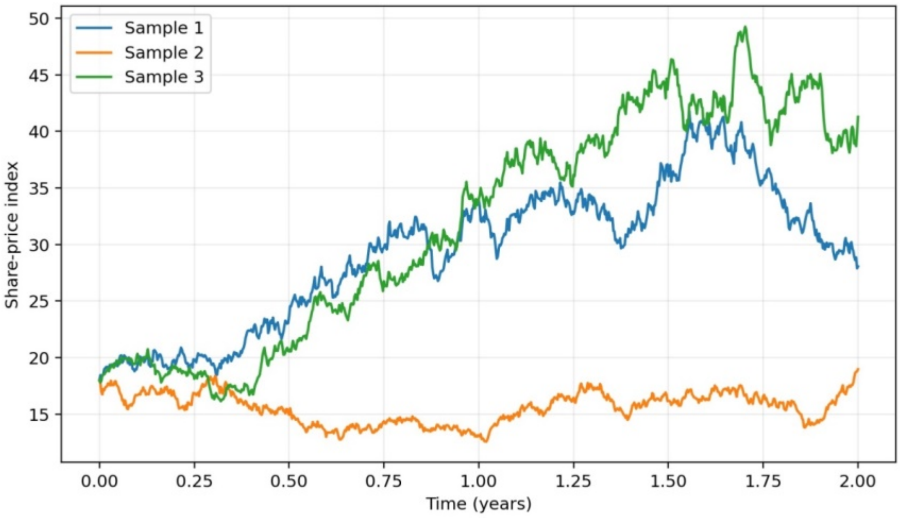

Figure 2: Illustrative sample paths from the exponential-growth GBM.

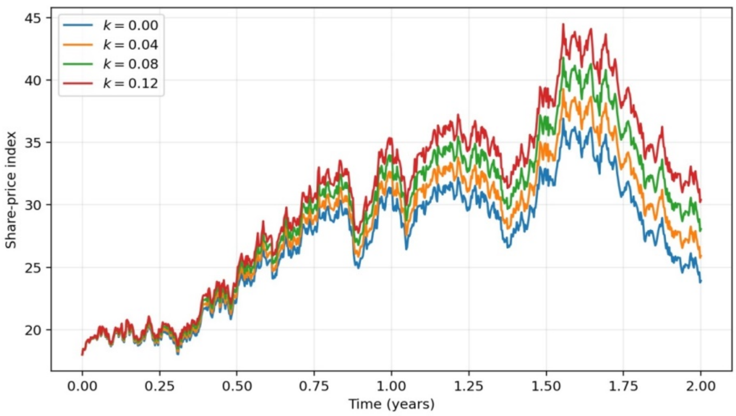

Figure 3: Sensitivity of the simulated price to the growth-adjustment parameter k.

and absolute variance, but the coefficient of variation $\sqrt{e^{\sigma^2 t}-1}$ is independent of k. It is therefore inaccurate to claim that k alone increases relative uncertainty.

For risk management, the model can be used to compare scenarios by varying k, μ, and σ. However, the figures should be interpreted as demonstrations of model behavior. No bank-level data, parameter-estimation procedure, goodness-of-fit test, or out-of-sample validation is supplied in the present study. Accordingly, the results do not establish that the model accurately describes Access Bank, First Bank, or any other specific institution.

4. Conclusion

The growth-adjusted GBM provides a transparent framework for analyzing bank valuation under long-term growth, inflation, and market noise. Its explicit solution is positive and unique, and the sign of $\mu + k$ determines whether the expected price declines, remains stable, or grows. Volatility permits individual paths to depart substantially from the expected trajectory.

The adjustment $k = g - \pi$ separates the intended growth and inflation effects, although only their net value is identifiable from this specification without additional information. Future work should estimate g, π, μ, and σ from Nigerian bank and macroeconomic data, report simulation parameters and random seeds, and evaluate forecasting performance out of sample.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability

No external dataset was analyzed in this theoretical and illustrative study. The mathematical derivations and simulation specification required to reproduce the model are provided in the article.

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