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Bounded Relative Distances of Symmetric Prime Pairs Around Even Integers

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Abstract

This paper investigates the distribution of symmetric prime pairs around the midpoint of even integers. For a given even integer $2n$, prime pairs (p_1, p_2) symmetrically positioned about n are considered, satisfying

$$n - p_1 = p_2 - n,$$

or equivalently $p_1 + p_2 = 2n$. Particular attention is given to the nearest symmetric prime pair associated with each even integer. Using the symmetric distance $d = |n - p_i|$, $i = 1, 2$, the relative distance ratio

$$R_n = \frac{d}{n}$$

is introduced as a normalized measure of how far the nearest symmetric prime pair lies from the midpoint. Computational experiments were carried out for selected even integers up to 10^6. The resulting data show bounded behavior within the tested range and suggest an empirical asymptotic decay of R_n toward zero as n increases, except in cases where the midpoint itself is prime, for which $R_n = 0$. Within the computed data, the largest observed ratio occurs at $2n = 44$, where $R_n = 9/22 = 0.4090909\ldots$. The paper presents a ratio-based framework for studying local symmetric prime distributions and highlights open questions concerning upper bounds, asymptotic estimates, and additional symmetric prime pairs farther from the midpoint.

Keywords: symmetric prime pairs; prime distribution; Goldbach representations; relative distance ratio; asymptotic decay; computational number theory.

1. Introduction

Prime numbers and their distribution remain central themes in number theory because of their structural importance and their applications in computation, cryptography, and discrete mathematics. Despite extensive study, several questions about the distribution of primes remain open. One of the most famous of these is Goldbach's conjecture, which asserts that every even integer greater than two can be expressed as the sum of two primes.

For an even integer $2n$, any representation

$$2n = p_1 + p_2$$

may be interpreted geometrically as a symmetric arrangement around the midpoint n. Indeed, the equality above is equivalent to

$$n - p_1 = p_2 - n.$$

Thus, the primes p_1 and p_2 are equidistant from n. This observation motivates the study of symmetric prime pairs around even integers.

Previous works have considered several aspects of prime gaps, extremal behavior, and symmetric prime distributions. Pommerance studied prime graphs and extremal properties of prime numbers, while later authors investigated symmetric distributions and related gap behavior. The present paper focuses on the nearest symmetric prime pair around the midpoint of a given even integer and introduces a normalized ratio measuring the relative distance between the midpoint and such a pair.

Let d_n denote the least nonnegative distance from n to a pair of primes symmetrically placed around n. The main quantity studied in this work is

$$R_n = \frac{d_n}{n}.$$

This ratio compares the distance of the nearest symmetric prime pair with the size of the midpoint itself. Computational evidence suggests that this ratio remains bounded and becomes small for large values of n.

The objectives of this paper are to define and study the relative distance ratio associated with symmetric prime pairs, to compute its values for selected even integers up to 10^6, to present numerical evidence of its boundedness and decay, and to formulate conjectures and open problems suggested by the computations.

2. Preliminaries and Definitions

Definition 2.1 (Symmetric prime pair)**.** Let $2n$ be an even integer with $n \geq 2$. A pair of primes (p_1, p_2) is called a symmetric prime pair about n if

$$p_1 + p_2 = 2n.$$

Equivalently,

$$n - p_1 = p_2 - n.$$

Definition 2.2 (Nearest symmetric prime pair)**.** For a given even integer $2n$, a symmetric prime pair (p_1, p_2) is called a nearest symmetric prime pair if the common distance from n is minimal among all symmetric prime pairs associated with $2n$.

Definition 2.3 (Symmetric prime distance)**.** Let (p_1, p_2) be a symmetric prime pair about n. The symmetric prime distance is

$$d = |n - p_1| = |p_2 - n|.$$

Definition 2.4 (Relative distance ratio)**.** For a nearest symmetric prime pair associated with $2n$, define

$$R_n = \frac{d_n}{n},$$

where d_n is the least symmetric prime distance. The value R_n measures the normalized distance of the nearest symmetric prime pair from the midpoint.

Theorem 2.5 (Euclid)**.** *There are infinitely many prime numbers.*

Theorem 2.6 (Prime Number Theorem)**.** *Let $\pi(x)$ denote the number of primes not exceeding x. Then*

$$\pi(x) \sim \frac{x}{\log x}$$

as $x \to \infty$.

Conjecture 2.7 (Goldbach)**.** *Every even integer greater than two can be expressed as the sum of two primes.*

Corollary 2.8. *If Goldbach's conjecture holds for $2n$, then n is the midpoint of at least one symmetric prime pair.*

Proposition 2.9. *If n is prime, then $R_n = 0$.*

Proof. If n is prime, then (n, n) is a symmetric prime pair about n because $n + n = 2n$. Therefore $d_n = 0$, and hence $R_n = d_n/n = 0$. □

Proposition 2.10. *For every even integer $2n$ that has a Goldbach representation, the corresponding relative distance ratio satisfies*

$$0 \leq R_n < 1.$$

Proof. If $2n = p_1 + p_2$ for primes p_1, p_2, then the nearest symmetric prime distance d_n is nonnegative and is less than n for a nontrivial representation. Hence $0 \leq d_n < n$, and division by n gives $0 \leq R_n < 1$. □

3. Computational Methodology

For each even integer $2n$ under consideration, the midpoint n was computed. A deterministic search was then performed to identify the smallest nonnegative integer k such that both

$$n - k \quad \text{and} \quad n + k$$

are prime. Once such a value of k was found, the nearest symmetric prime pair was recorded as

$$p_1 = n - k, \qquad p_2 = n + k,$$

and the relative distance ratio was computed as

$$R_n = \frac{k}{n}.$$

The computational procedure may be summarized as follows:

1. Choose an even integer $2n$ and compute its midpoint n.
2. Set $k = 0$.
3. Test whether $n - k$ and $n + k$ are both prime.
4. If both are prime, record $p_1 = n - k$, $p_2 = n + k$, $d_n = k$, and $R_n = k/n$.
5. If not, increase k by one and repeat the primality test.

This algorithm was implemented using Python with deterministic primality checking. The resulting numerical data were tabulated for analysis.

4. Numerical Results and Observations

Table 1 presents the numerical values extracted from the computational study. The columns give the even integer $2n$, the midpoint n, the nearest symmetric prime pair, the distance ratio written as $d : n$, and the corresponding relative distance ratio R_n.

The data show several consistent patterns. First, whenever the midpoint n is prime, the ratio is zero. Second, the observed values remain bounded throughout the tested range. Third, although local fluctuations occur, the ratios become very small for large values of $2n$. In the data presented, the largest observed value is

$$R_22 = \frac{9}{22} = 0.4090909\ldots,$$

which occurs at $2n = 44$.

Table 1: Nearest symmetric prime pairs and relative distance ratios.

Even integer $2n$	Midpoint n	Nearest prime pair (p_1, p_2)	Distance $d : n$	Relative ratio R_n
4	2	2,2	0:2	0
6	3	3,3	0:3	0
8	4	3,5	1:4	0.25
10	5	5,5	0:5	0.2
12	6	5,7	1:6	0.166666
14	7	7,7	0:7	0
16	8	5,11	3:8	0.375
18	9	7,11	2:9	0.2222
20	10	7,13	3:10	0.3
22	11	11,11	0:11	0
24	12	11,13	1:12	0.083333
26	13	13,13	0:13	0
28	14	11,17	3:14	0.21428
30	15	13,17	2:15	0.133333
32	16	13,19	3:16	0.1875
34	17	17,17	0:17	0
36	18	17,19	1:18	0.055555
38	19	19,19	0:19	0
40	20	17,23	3:20	0.15
42	21	19,23	2:21	0.095238
44	22	13,31	9:22	0.40909090
46	23	17,29	6:23	0.2608695652
48	24	17,31	7:24	0.2916666666
50	25	19,31	6:25	0.24
52	26	23,29	3:26	0.115384615
54	27	23,31	4:27	0.1481481
56	28	19,37	9:28	0.3214285714
58	29	29,29	0:29	0
60	30	29,31	1:30	0.0333333
62	31	31,31	0:31	0
64	32	23,41	9:32	0.28125
66	33	29,37	4:33	0.12121212
68	34	31,37	3:34	0.08823529
70	35	29,41	4:35	0.1142857
72	36	31, 41	5:36	0.13888888
74	37	37, 37	0:37	0
76	38	29, 47	9:38	0.23684211
78	39	37, 41	2:39	0.05128205
80	40	37, 43	3:40	0.075
82	41	41, 41	0:41	0
84	42	41, 41	1:42	0.023809523809

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Even integer $2n$	Midpoint n	Nearest prime pair (p_1, p_2)	Distance $d : n$	Relative ratio R_n
86	43	43, 43	0:43	0
88	44	41, 47	3:44	0.0688181
90	45	43, 47	2:45	0.044444444
92	46	43, 49	3:46	0.0652173913
94	47	47, 47	0:47	0
96	48	47, 49	1:48	0.02083333333
98	49	49, 49	0:49	0
100	50	47, 53	3:50	0.06
102	51	43, 59	8:51	0.156862745
104	52	43, 61	9:52	0.1730769231
106	53	53, 53	0:53	0
108	54	47, 61	7:54	0.129629629
110	55	47, 63	8:55	0.145454545
112	56	53, 59	3:56	0.053571428
114	57	53, 61	4:57	0.070175386
116	58	43, 73	15:58	0.2586206897
118	59	59, 59	0:59	0
120	60	59, 61	1:60	0.0166666
122	61	61, 61	0:61	0
124	62	53, 71	9:62	0.14516129
126	63	59, 67	4:63	0.063492063
128	64	61, 67	3:64	0.046875
130	65	59, 71	6:65	0.0923076923
132	66	61, 71	5:66	0.075757575
134	67	61, 73	6:67	0.0895522388
136	68	67, 71	1:68	0.01470588
138	69	67, 73	3:69	0.043478260869
140	70	67, 73	3:70	0.042857
142	71	59,83	12:71	0.16901408
144	72	71,73	1:72	0.01388888
146	73	73,73	0:73	0
148	74	59,89	15:74	0.20270270
150	75	71,79	4:75	0.053333
152	76	73,79	3:76	0.039473684
154	77	71,83	6:77	0.077922077
156	78	73,83	5:78	0.06410356
158	79	79,79	0:79	0
160	80	71,89	9:80	0.1125
162	81	79,83	2:81	0.024691358
164	82	67,97	15:82	0.18292682926
166	83	83,83	0:83	0
228	114	101,127	13:114	0.1140350
584	294	281,307	13:294	0.044217687

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Even integer $2n$	Midpoint n	Nearest prime pair (p_1, p_2)	Distance $d : n$	Relative ratio R_n
636	318	283,353	35:318	0.110062893
1240	620	599,641	21:620	0.0338709677
1672	836	809,863	27:836	0.03229665072
1680	840	827,853	13:840	0.01547619048
8004	4002	4001,4003	1:4002	0.0002498750725
8022	4011	4003,4019	8:4011	0.001994515084
9860	4930	4903,4957	27:4930	0.00547663428
10,000	5000	4989,5011	11:5000	0.0022
10214	5107	5107,5017	0:5107	0
17370	8685	8677,8693	8:8685	0.0009211283823
17902	8951	8933,8969	18:8951	0.002010948497
18970	9485	9473,9497	12:9485	0.001265155086
19352	9676	9631,9721	45:9676	0.0045606821
19354	9677	9677,9677	0:9677	0
19356	9678	9623,9733	55:9678	0.00568299
20000	10000	9931,10069	69:10000	0.0069
20012	10006	9973,10006	33:10006	0.0032980211872
20014	10007	10007,10007	0:10007	0
20016	10008	10007,10009	1:10008	0.00009992
20018	10009	10009,10009	0:10009	0
20020	10010	9941,10079	69:10010	0.00689310
20128	10064	10037,10091	27:10064	0.002682829888
22492	11246	11213,11279	33:11247	0.0029341157642
30000	15000	14983,15017	17:15000	0,001133333333
30156	15078	15073,15083	5:15078	0.0003316089667
40000	20000	19979,20021	21:20000	0.00105
40148	20074	20047,20101	27:20074	0.0013450234133
40158	20079	20051,20107	28:20079	0.0013944917575
43134	21567	21557,21577	10:21567	0.0004636713497
50000	25000	24967,25033	33:2500	0.0132
54022	27011	27011,27011	0:27011	0
54024	27012	26993,27031	19:27012	0.0007034171263
60000	30000	29989,30011	11:30000	0.000366666
60770	30385	30367,30403	18:30385	0.0005923975645
62066	31033	31033,31033	0:31033	0
62068	31034	30977,31091	57:31034	0.0018366952374
70000	35000	34981,35019	19:35000	0.000542871428
80000	40000	39937,40063	63:40000	0.001575
90000	45000	44987,45013	13:45000	0.000288888888
100000	50000	49877,50123	120:50000	0.0024
200000	100000	99989,100019	19:100000	0.00019
300000	150000	149713,150287	287:150000	0.001913333
400000	200000	199963,200033	33:200000	0.000165

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Even integer $2n$	Midpoint n	Nearest prime pair (p_1, p_2)	Distance $d : n$	Relative ratio R_n
500000	250000	249973,250027	27:250000	0.000108
600000	300000	299993,300007	7:300000	0.00002333333333
700000	350000	349967,350033	33:350000	0.0000942857142857
724198	362099	362099,362099	0:362099	0
780896	390448	390433,390463	15: 390448	0.000038417407695
800000	400000	399913,400087	87:400000	0.0002175
814586	407293	407287,407299	6:407293	0.0000147314095749
844622	422311	422311,422311	0:422311	0
900000	450000	449989,450011	11:450000	0.0000244444444444
920124	460062	460013,460111	49:460062	0.0001065073837
944842	472421	472421, 472421	0: 472421	0
1000000	500000	499943,500057	57:500000	0.000114

5. Heuristic Analysis and Main Conjecture

The numerical behavior suggests that the nearest symmetric prime pair becomes proportionally closer to the midpoint as n increases. This can be motivated heuristically using the Prime Number Theorem. Near a large integer n, the probability that an integer is prime is approximately

$$\frac{1}{\log n}.$$

Under a Cramer-type independence assumption, the probability that both $n-k$ and $n+k$ are prime is roughly

$$\frac{1}{(\log n)^2}.$$

Thus, the expected waiting distance to the first symmetric prime pair may be modeled as being of order $(\log n)^2$. This suggests

$$d_n \approx (\log n)^2,$$

and consequently

$$R_n = \frac{d_n}{n} \approx \frac{(\log n)^2}{n}.$$

Since $(\log n)^2/n \to 0$ as $n \to \infty$, the heuristic supports the observed decay of R_n.

Conjecture 5.1 (Asymptotic decay of the relative distance ratio)**.** *Let d_n denote the distance from n to the nearest symmetric prime pair about n, and let*

$$R_n = \frac{d_n}{n}.$$

Then

$$\lim_{n \to \infty} R_n = 0.$$

Equivalently,

$$d_n = o(n).$$

Remark 5.2. The conjecture is supported by the computational data presented in Table 1. However, a proof would require deep information about prime gaps and Goldbach-type representations. Therefore, the statement is presented as a computationally motivated conjecture rather than a theorem.

6. Additional Symmetric Prime Pairs

In addition to the nearest symmetric prime pair, one may consider further prime pairs located at greater symmetric distances from the midpoint. The original computations also considered residue-class patterns related to the fact that primes greater than 3 are congruent to 1 or 5 modulo 6. Table 2 reproduces the residue-pattern table from the manuscript as a guide for further investigation.

Table 2: Residue-pattern table associated with symmetric arrangements modulo 6.

↓						↓					
1	0	5	4	3	2	1	0	5	4	3	...
1	2	3	4	5	0	1	2	3	4	5	...
			↓						↓		
2	1	0	5	4	3	2	1	0	5	4	...
2	3	4	5	0	1	2	3	4	5	0	...
		↓		↓				↓		↓	
3	2	1	0	5	4	3	2	1	0	5	...
3	4	5	0	1	2	3	4	5	0	1	...
			↓						↓		
4	3	2	1	0	5	4	3	2	1	0	...
4	5	0	1	2	3	4	5	0	1	2	...
↓						↓					
5	4	3	2	1	0	5	4	3	2	1	...
5	0	1	2	3	4	5	0	1	2	3	...
	↓				↓		↓				
0	5	4	3	2	1	0	5	4	3	2	...
0	1	2	3	4	5	0	1	2	3	4	...

The probability of obtaining additional pairs (p_1, p_2) farther from the midpoint while preserving $p_1 + p_2 = 2n$ remains an open problem. A natural extension of the present work is to study the second-nearest and third-nearest symmetric prime pairs and compare their normalized distances with R_n.

7. Open Problems

The computations lead to several open questions:

1. Does there exist a universal constant $C < 1$ such that $R_n \leq C$ for all even integers $2n$ satisfying Goldbach's conjecture?

2. Is the observed maximum value $9/22$ at $2n = 44$ the global maximum of R_n?

3. Can one prove that $R_n \to 0$ as $n \to \infty$?

4. What is the correct average order of d_n?

5. How do the second-nearest and third-nearest symmetric prime pairs behave asymptotically?

8. Conclusion

This paper introduced the relative distance ratio

$$R_n = \frac{d_n}{n}$$

for studying nearest symmetric prime pairs around the midpoint of even integers. Computational results for selected even integers up to 10^6 suggest that the ratio remains bounded and tends to decrease as n increases. The largest observed ratio in the data occurs at $2n = 44$, with value $9/22 = 0.4090909\ldots$. Whenever n is prime, the ratio is zero because (n, n) itself is a symmetric prime pair.

The results do not constitute a proof of Goldbach's conjecture. Rather, they provide a normalized framework for examining the local behavior of Goldbach representations. The observed asymptotic decay motivates further theoretical study of upper bounds, growth rates of nearest symmetric prime distances, and the distribution of additional symmetric prime pairs.

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Conflict of Interest

The authors declare no conflict of interest.

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