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Stochastic System Estimates to Assess Corporate Investor Portfolio Value in Stock Markets

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Abstract

Stock-market uncertainty creates significant challenges in the assessment of corporate investment portfolio values, particularly where expected returns, intrinsic growth rates, interest rates, volatility, and random market movements interact over time. This study develops a system of stochastic differential equations for estimating the wealth dynamics of three corporate investors under such market conditions. Analytical solutions are obtained through Itô's lemma by applying a logarithmic transformation to each wealth process, leading to explicit stochastic expressions for portfolio-value evolution. Numerical evaluations of the model parameters show that higher intrinsic growth rates increase terminal wealth, whereas higher interest-rate parameters reduce portfolio values. Volatility affects both the deterministic Itô correction and the stochastic component of the wealth processes, thereby influencing the distribution and sensitivity of terminal portfolio values. Under the parameter values considered, the second corporate investor records the highest wealth estimates among the three investors. The proposed stochastic framework therefore provides a quantitative basis for assessing corporate-investor portfolio values and for examining the effects of key financial parameters under uncertain stock-market conditions.

Keywords: wealth; stochastic systems; investments; interest rates; stock prices.

1. Introduction

Investment decisions are inherently associated with risk. Variations in stock and portfolio returns make risk a central consideration in portfolio management and provide investors with a mathematical basis for comparing alternative decisions. Financial securities such as bonds, stocks, and property are exposed to varying degrees of risk. In practice, risk transfer and risk sharing are also important in financial management, particularly in insurance and reinsurance arrangements.

Trading involves the transfer of a security from a seller to a buyer at an agreed price. Stock-exchange trading may occur on a physical trading floor or through an electronic network. Investors rely on market indicators and model parameters to guide investment decisions and improve expected returns. Equity returns may arise from capital gains, when a stock is resold at a price above its purchase price, and from dividends distributed to shareholders. A useful mathematical description of such market quantities is through diffusion processes, particularly geometric Brownian motion, which is widely used as a reference model for financial variables [2].

Several authors have developed stochastic models for stock-market prices and investment values. Azor, Nwobi, and Amadi [3] studied systems of stochastic differential equations (SDEs) for economic investments with rate-of-return and asset-valuation dynamics involving price-index, periodic, and multiplicative effects. Amadi, Tamunotonye, and Azor [4] examined asset-value models with a delay parameter and established conditions governing asset values. Adeosun, Edeki, and Ugbebor [1] analyzed the stochastic behaviour of stock-market price models and reported that their proposed formulation was useful for stock-price prediction. Ofomata et al. [5] considered selected stocks in the Nigerian Stock Exchange and estimated drift and volatility coefficients, while Dmouj [6] discussed geometric Brownian motion for stock-price modelling. Related stochastic approaches to stock-price behaviour and capital-market dynamics appear in [7-14] and [15-22].

The aim of this study is to develop estimates for three stochastic systems that measure the wealth of corporate investors in stock markets. Ekakaa, Nwobi, and Amadi [15] considered interacting investors using a Lotka-Volterra-type competition model. The present formulation instead incorporates stochastic terms and models the effects of intrinsic growth rates, interest rates, periodic wealth effects, and stock volatility on the wealth of each investor. The remainder of the paper presents the mathematical formulation, analytical solution, simulation results, and conclusions.

2. Materials and Methods

2.1. Mathematical Framework

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space and let $W(t) = (W_1(t), W_2(t), \dots, W_m(t))^\top$ be an m-dimensional Brownian motion. A general stochastic differential equation may be written as

$$dX(t) = f(t, X(t))\, dt + g(t, X(t))\, dW(t), \qquad 0 \leq t \leq T, \tag{2.1}$$

with initial condition

$$X(0) = X_0. \tag{2.2}$$

Equivalently,

$$X(t) = X_0 + \int_0^t f(s, X(s))\, ds + \int_0^t g(s, X(s))\, dW(s). \tag{2.3}$$

Itô's lemma. Let $F(S, t)$ be twice continuously differentiable in S and once differentiable in t, and suppose

$$dS_t = a_t\, dt + b_t\, dW_t. \tag{2.4}$$

Then, retaining the terms that survive in the Itô calculus limit,

$$dF = \left(a_t \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} b_t^2 \frac{\partial^2 F}{\partial S^2} \right) dt + b_t \frac{\partial F}{\partial S} \, dW_t. \tag{2.5}$$

For a geometric Brownian motion $dS = \mu S \, dt + \sigma S \, dW_t$, this becomes

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} \, dW_t. \tag{2.6}$$

2.2. Problem Formulation

Let the wealth vector of the three corporate investors be

$$V(t) = \big(V_1(t), V_2(t), V_3(t)\big)^\top.$$

The model assumes that the rate of change of investor wealth depends on expected return, intrinsic growth, interest-rate effects, stock volatility, and stochastic market fluctuations. The governing system is

$$dV_1(t) = (\mu\alpha_1 - \beta_1) V_1(t) \, dt + \sigma V_1(t) \, dW_t^1, \tag{2.7}$$
$$dV_2(t) = (\mu\alpha_2 - \beta_2) V_2(t) \, dt + \sigma V_2(t) \, dW_t^2, \tag{2.8}$$
$$dV_3(t) = (\mu\alpha_3 - \beta_3) V_3(t) \, dt + \sigma V_3(t) \, dW_t^3, \tag{2.9}$$

subject to

$$V_1(0) = V_{10} > 0, \qquad V_2(0) = V_{20} > 0, \qquad V_3(0) = V_{30} > 0. \tag{2.10}$$

Here μ denotes the expected stock return, σ is stock volatility, W_t^i is a Wiener process for investor i, α_i represents the intrinsic growth-rate parameter, and β_i represents the corresponding interest-rate parameter.

2.3. Method of Solution

For the first investor, let $f(V_1(t), t) = \ln V_1(t)$. Then

$$\frac{\partial f}{\partial V_1} = \frac{1}{V_1}, \qquad \frac{\partial^2 f}{\partial V_1^2} = -\frac{1}{V_1^2}, \qquad \frac{\partial f}{\partial t} = 0. \tag{2.11}$$

Applying Itô's lemma to (2.7) gives

$$d \ln V_1(t) = \left(\mu\alpha_1 - \beta_1 - \frac{1}{2}\sigma^2 \right) dt + \sigma \, dW_t^1. \tag{2.12}$$

Integrating from 0 to t,

$$\ln \frac{V_1(t)}{V_{10}} = \left(\mu\alpha_1 - \beta_1 - \frac{1}{2}\sigma^2 \right) t + \sigma W_t^1, \tag{2.13}$$

so that

$$V_1(t) = V_{10} \exp\left[\left(\mu\alpha_1 - \beta_1 - \frac{1}{2}\sigma^2 \right) t + \sigma W_t^1 \right]. \tag{2.14}$$

Following the same procedure for the second and third investors yields

$$V_2(t) = V_{20} \exp\left[\left(\mu\alpha_2 - \beta_2 - \frac{1}{2}\sigma^2\right) t + \sigma W_t^2\right], \tag{2.15}$$

and

$$V_3(t) = V_{30} \exp\left[\left(\mu\alpha_3 - \beta_3 - \frac{1}{2}\sigma^2\right) t + \sigma W_t^3\right]. \tag{2.16}$$

Equations (2.14)–(2.16) provide the stochastic wealth estimates used in the numerical study.

3. Results and Discussion

The source study reports simulation parameters including $\alpha_1 = 0.0370$, $\alpha_2 = 0.0300$, $\alpha_3 = 0.0280$, $\alpha_4 = 0.0260$, $\alpha_5 = 0.0360$, $\beta_1 = 0.0014$, $\beta_2 = 0.0010$, $\beta_3 = 0.0008$, $\mu = 0.25$, $K = 1$, $h = 25$, $\sigma = 0.03$, $t = 5$, and unit Wiener-process realizations, with reported initial values including $V_{10} = 60.77$, $V_{20} = 50.25$, and $V_{30} = 40.10$. The tables below reproduce the numerical values reported in the manuscript.

Table 1: Effect of intrinsic growth rate on the wealth of the first corporate investor; $\mu = 0.25$, $\beta_1 = 0.0014$.

V_{10}	α_1	σ	t	$V_1(t)$	α_1	$V_1(t)$
60.77	0.1	0.20000	5.0000	75.5728	1.1	215.9607
	0.2	0.20000	5.0000	85.6352	1.2	298.8962
	0.3	0.20000	5.0000	107.2429	1.3	338.6937
	0.4	0.20000	5.0000	134.3027	1.4	383.7903
50.25	0.1	0.20000	5.0000	62.4903	1.1	218.1124
	0.2	0.20000	5.0000	70.8107	1.2	247.1537
	0.3	0.20000	5.0000	80.2391	1.3	280.0619
	0.4	0.20000	5.0000	90.9228	1.4	317.3516
40.10	0.1	0.20000	5.0000	49.8678	1.1	174.05587
	0.2	0.20000	5.0000	56.5077	1.2	197.2311
	0.3	0.20000	5.0000	64.0316	1.3	223.4922
	0.4	0.20000	5.0000	72.5573	1.4	253.2498
36.0000	0.1	0.20000	5.0000	44.7691	1.1	156.2596
	0.2	0.20000	5.0000	50.7301	1.2	177.0654
	0.3	0.20000	5.0000	57.4847	1.3	200.6413
	0.4	0.20000	5.0000	65.1387	1.4	227.3564

Table 2: Effect of intrinsic growth rate on the wealth of the second corporate investor; $\mu = 0.75,\ \beta_2 = 0.0010$.

V_{20}	α_2	σ	t	$V_2(t)$	α_2	$V_2(t)$
5.0000	0.1	0.20000	5.0000	7.9999	1.1	340.1674
	0.2	0.20000	5.0000	11.6399	1.2	494.9407
	0.3	0.20000	5.0000	16.9359	1.3	720.1344
	0.4	0.20000	5.0000	24.6416	1.4	1001.6841
6.0000	0.1	0.20000	5.0000	9.5999	1.1	408.2009
	0.2	0.20000	5.0000	13.9679	1.2	593.9288
	0.3	0.20000	5.0000	20.3231	1.3	864.1613
	0.4	0.20000	5.0000	29.5699	1.4	1202.0209
5.7000	0.1	0.20000	5.0000	2.679	1.1	387.7909
	0.2	0.20000	5.0000	13.2695	1.2	564.2324
	0.3	0.20000	5.0000	19.3070	1.3	820.9533
	0.4	0.20000	5.0000	28.09148	1.4	1141.9198
5.2000	0.1	0.20000	5.0000	8.3199	1.1	353.7741
	0.2	0.20000	5.0000	12.1055	1.2	514.7383
	0.3	0.20000	5.0000	17.6134	1.3	748.9398
	0.4	0.20000	5.0000	25.6273	1.4	1041.7514

Table 3: Effect of intrinsic growth rate on the wealth of the third corporate investor; $\mu = 0.75,\ \beta_3 = 0.0008$.

V_{30}	α_3	σ	t	$V_3(t)$	α_3	$V_3(t)$
5.0000	0.1	0.20000	5.0000	8.0080	1.1	340.5078
	0.2	0.20000	5.0000	11.6515	1.2	495.4359
	0.3	0.20000	5.0000	16.9529	1.3	720.8549
	0.4	0.20000	5.0000	24.6663	1.4	1048.8377
6.0000	0.1	0.20000	5.0000	9.6096	1.1	408.6093
	0.2	0.20000	5.0000	13.9818	1.2	594.5230
	0.3	0.20000	5.0000	20.3435	1.3	865.0259
	0.4	0.20000	5.0000	29.5996	1.4	1258.6053
5.7000	0.1	0.20000	5.0000	9.1291	1.1	388.1788
	0.2	0.20000	5.0000	13.2827	1.2	564.7969
	0.3	0.20000	5.0000	19.3263	1.3	821.7746
	0.4	0.20000	5.0000	28.1196	1.4	1195.6750
5.2000	0.1	0.20000	5.0000	8.3283	1.1	354.1281
	0.2	0.20000	5.0000	12.1176	1.2	515.2533
	0.3	0.20000	5.0000	17.6309	1.3	749.6891
	0.4	0.20000	5.0000	25.6530	1.4	1090.7912

Table 4: Effect of interest rate on the wealth of the first corporate investor; $\mu = 0.25,\ \alpha_1 = 0.0370,\ \sigma = 0.2$.

V_{10}	β_1	α_1	t	$V_1(t)$	β_1	$V_1(t)$
60.77	0.1	0.0370	5.0000	42.6636	0.8	1.2883
	0.2	0.0370	5.0000	25.8768	0.6	3.5020
	0.3	0.0370	5.0000	15.6951	0.4	9.5195
	0.4	0.0370	5.0000	9.5195	0.2	25.8768
50.25	0.1	0.0370	5.0000	35.2780	0.8	1.0653
	0.2	0.0370	5.0000	21.3972	0.6	2.8958
	0.3	0.0370	5.0000	12.9781	0.4	7.8716
	0.4	0.0370	5.0000	7.8716	0.2	21.3972
40.10	0.1	0.0370	5.0000	28.1522	0.8	0.8501
	0.2	0.0370	5.0000	17.0752	0.6	2.3109
	0.3	0.0370	5.0000	10.3566	0.4	6.2816
	0.4	0.0370	5.0000	6.2816	0.2	17.0752
36.0000	0.1	0.0370	5.0000	25.2738	0.8	0.7632
	0.2	0.0370	5.0000	15.3293	0.6	2.0746
	0.3	0.0370	5.0000	9.2977	0.4	5.6394
	0.4	0.0370	5.0000	5.6394	0.2	15.3293

Table 5: Effect of interest rate on the wealth of the second corporate investor; $\mu = 0.75,\ \alpha_2 = 0.0300$.

V_{20}	β_2	α_2	t	$V_2(t)$	β_2	$V_2(t)$
5.0000	0.1	0.030000	5.0000	3.7507	0.8	0.1133
	0.2	0.030000	5.0000	2.2750	0.6	0.3079
	0.3	0.030000	5.0000	1.3800	0.4	0.837
	0.4	0.030000	5.0000	0.8370	0.2	2.2750
6.0000	0.1	0.030000	5.0000	4.5006	0.8	0.1359
	0.2	0.030000	5.0000	2.7300	0.6	0.3694
	0.3	0.030000	5.0000	1.6560	0.4	1.0044
	0.4	0.030000	5.0000	1.0044	0.2	2.73
5.7000	0.1	0.030000	5.0000	4.2756	0.8	0.1291
	0.2	0.030000	5.0000	2.5935	0.6	0.3510
	0.3	0.030000	5.0000	1.5732	0.4	0.9542
	0.4	0.030000	5.0000	0.9542	0.2	2.5935
5.2000	0.1	0.030000	5.0000	3.9005	0.8	0.1178
	0.2	0.030000	5.0000	2.3660	0.6	0.3202
	0.3	0.030000	5.0000	1.4352	0.4	0.8705
	0.4	0.030000	5.0000	0.8705	0.2	2.366

Table 6: Effect of interest rate on the wealth of the third corporate investor; $\mu = 0.75,\ \alpha_3 = 0.0280$.

V_{30}	β_3	α_3	t	$V_3(t)$	β_3	$V_3(t)$
5.0000	0.1	0.030000	5.0000	3.7227	0.8	0.1124
	0.2	0.030000	5.0000	2.2579	0.6	0.3056
	0.3	0.030000	5.0000	1.3695	0.4	0.8306
	0.4	0.030000	5.0000	0.8306	0.2	2.2579
6.0000	0.1	0.030000	5.0000	4.4672	0.8	0.1349
	0.2	0.030000	5.0000	2.7095	0.6	0.3667
	0.3	0.030000	5.0000	1.6434	0.4	0.9968
	0.4	0.030000	5.0000	0.9968	0.2	2.7095
5.7000	0.1	0.030000	5.0000	4.2438	0.8	0.1282
	0.2	0.030000	5.0000	2.5740	0.6	0.3484
	0.3	0.030000	5.0000	1.5612	0.4	0.9469
	0.4	0.030000	5.0000	0.9469	0.2	2.5740
5.2000	0.1	0.030000	5.0000	3.6716	0.8	0.1169
	0.2	0.030000	5.0000	2.3482	0.6	0.3178
	0.3	0.030000	5.0000	1.4243	0.4	0.8639
	0.4	0.030000	5.0000	0.8639	0.2	2.3482

Tables 1–3 show the effect of changes in the intrinsic growth-rate parameters on investor wealth. In the reported simulations, larger intrinsic growth-rate values are associated with larger terminal wealth values. This is consistent with the exponential form of (2.14)–(2.16), in which an increase in α_i, all else being equal, increases the deterministic component of the exponent.

Tables 4–6 show the effect of interest-rate parameters on wealth. As the relevant β_i increases, the wealth values decrease because β_i enters the drift component with a negative sign. Economically, higher discount or financing rates reduce the present value of future cash flows. Conversely, lower rates raise the present value of those cash flows. The stochastic term also implies that volatility affects the distribution of terminal wealth, and the Itô correction $-\frac{1}{2}\sigma^2$ appears directly in the logarithmic wealth dynamics.

Across the reported numerical comparisons, the second corporate investor attains the largest wealth values in the portfolio simulations. This may reflect the combination of the selected initial wealth, intrinsic-growth parameter, interest-rate parameter, and other model assumptions. The relative ranking of investors can nevertheless change when market conditions, portfolio composition, or parameter values are altered.

4. Conclusion

This study formulates three stochastic differential equations for assessing corporate-investor portfolio wealth in a stock-market setting and obtains analytical solutions using Itô's lemma. The resulting solutions show explicitly how expected returns, intrinsic growth rates, interest-rate parameters, and volatility affect the stochastic evolution of wealth. The reported numerical results indicate that higher intrinsic growth rates increase wealth, higher interest-rate parameters reduce wealth, and volatility influences both the deterministic Itô correction and the stochastic dispersion of the wealth process. The simulations further show that the second corporate investor records the largest wealth values under the reported parameter settings. Future work may extend the model by allowing correlated Brownian motions, time-varying coefficients, transaction costs, portfolio rebalancing, and empirical calibration to observed market data.

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Conflict of Interest

The author declares no conflict of interest.

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