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Regime-Switching Dynamics of Nigerian Group Exchange Cement Equities: A Continuous-Time Markov Chain Analysis

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Abstract

We model daily price dynamics of BUA Cement and BZU Cement on the Nigerian Group Exchange (NGX) as three-state Continuous-Time Markov Chains (CTMCs) representing Low, Mid, and High regimes. Using nine months of data, estimated generator matrices show that BZU exhibits 2.4 times higher total transition intensity than BUA, with expected holding times of 1.5 months versus 3.0 months in bear states. Stationary distributions reveal that BUA spends 30% of time in the absorbing bull state, while BZU is uniformly distributed across regimes, indicating higher mean reversion. The CTMC results highlight BUA's relative stability compared with BZU's higher regime volatility. One-month transition forecasts show that BUA remains in the High regime with probability 100%, whereas BZU has only 51.3% probability of remaining High and a 28.3% chance of dropping directly to Low. These findings indicate that BUA is trend-persistent, while BZU is regime-unstable and more exposed to short-term downside switching risk.

Keywords: CTMCs; Stocks; Equity Prices; Stochastic Analysis; NGX; Regime-Switching.

1. Introduction

Equity prices in frontier markets such as the NGX frequently exhibit discontinuous regime shifts driven by exchange-rate policy, elections, inflationary pressure, sector-specific shocks, and investor expectations. Standard constant-parameter models, including the Black-Scholes geometric Brownian motion framework, often fail to capture persistent bull runs, rapid mean reversion, and regime clustering in sectoral equity prices. Stock price movements are influenced by macroeconomic conditions, market expectations, policy interventions, and global financial events [1]. Consequently, modelling stock market behaviour remains an important area in mathematical finance and applied stochastic analysis.

Continuous-Time Markov Chains (CTMCs) provide a tractable framework for representing price regimes and estimating transition rates between them. Unlike discrete-time models, CTMCs allow regime changes to occur at any time and are therefore useful when price movements are irregularly spaced. In finance, the states may represent volatility or price regimes such as Low, Mid, and High. The use of Markov chains in Nigerian stock market studies has been considered in relation to bank share price movement [2], finite-state stochastic analysis [3], principal component and Markov chain modelling [4], Oando stock variation assessment [5], capital market investment analysis [6], oil and gas stock modelling [7], expected return analysis [8], and Dangote Cement stock forecasting [9]. Related theoretical and applied models of asset pricing, credit risk, and stochastic market dynamics have also been developed in the broader literature [10, 12–14].

Despite these contributions, Nigerian banks and analysts often price equity risk using one-state models with fixed volatility calibrated within a single regime. However, BUA and BZU Cement show asymmetric regime persistence: BUA trends upward with an absorbing High state in the observed sample, while BZU mean-reverts rapidly. Standard one-state models may therefore understate risk by ignoring transition intensities and regime-dependent persistence. This paper estimates CTMC generator matrices for BUA and BZU Cement, computes stationary distributions, expected holding times, transition probability matrices, and graphical risk summaries. The study extends existing CTMC applications in Nigerian cement stocks [1] by comparing the structural differences between two cement equities under a three-state regime-switching framework.

The remainder of the paper is arranged as follows. Section 2 presents the methodology and mathematical formulation. Section 3 presents the results, discussion, and graphical analysis. Section 4 concludes the paper.

2. Methodology

Definition 2.1 (Markov chain)**.** *A stochastic process* $X = \{X_n : n \geq 0\}$ *is said to be a Markov chain if the Markov property is satisfied:*

$$P(X_{n+1} = j \mid X_0, X_1, \dots, X_n) = P(X_{n+1} = j \mid X_n), \tag{1}$$

for all $n \geq 0$ *and* $i, j \in S$*, where* S *is the state space.*

The Markov property in (1) implies that, for each $j \in S$,

$$P(X_{n+1} = j \mid X_{n_1}, X_{n_2}, \dots, X_{n_k}) = P(X_{n+1} = j \mid X_{n_k}), \tag{2}$$

for any $n_1 < n_2 < \cdots < n_k \leq n$. If $X_n = i$, the chain is said to be in state i at step n.

The one-step transition probability is

$$P(X_{n+1} = j \mid X_n = i). \tag{3}$$

Definition 2.2 (Homogeneous Markov chain)**.** *A Markov chain is homogeneous if*

$$P(X_{n+1} = j \mid X_n = i) = P(X_1 = j \mid X_0 = i), \tag{4}$$

for all n, i, j. The transition matrix $P = (p_{ij})$ is defined by

$$p_{ij} = P(X_{n+1} = j \mid X_n = i), \qquad p_{ij} \geq 0, \qquad \sum_j p_{ij} = 1. \tag{5}$$

2.1. Continuous-Time Markov Formulation

Let $N(t)$ denote the number of price-changing events up to time t. We assume that $N(t)$ follows a homogeneous Poisson process with rate $\lambda > 0$:

$$P(N(t+h) - N(t) = 1) = \lambda h + o(h), \qquad P(N(t+h) - N(t) \geq 2) = o(h). \tag{6}$$

The inter-arrival times $\tau_i = T_i - T_{i-1}$ are independently and identically distributed exponential random variables:

$$\tau_i \sim \text{Exp}(\lambda), \qquad E[\tau_i] = \frac{1}{\lambda}. \tag{7}$$

Let the finite state space be

$$S = \{s_1, s_2, \ldots, s_m\}. \tag{8}$$

Let X_n be the price state immediately after the nth jump. The Markov property gives

$$P(X_{n+1} = s_j \mid X_{n-1}, \ldots, X_n) = P(X_{n+1} = s_j \mid X_n = s_i) = p_{ij}. \tag{9}$$

The one-step transition matrix is

$$P = [p_{ij}]_{m \times m}, \qquad p_{ij} \geq 0, \qquad \sum_{j=1}^{m} p_{ij} = 1. \tag{10}$$

Define the continuous-time price process by

$$Y(t) = X_{N(t)}, \qquad t \geq 0. \tag{11}$$

Then $Y(t)$ is a CTMC with generator matrix $Q \in \mathbb{R}^{m \times m}$:

$$Q = \lambda(P - I). \tag{12}$$

Component-wise,

$$q_{ij} = \begin{cases} \lambda p_{ij}, & i \neq j, \\ -\lambda(1 - p_{ii}) = -\sum_{j \neq i} q_{ij}, & i = j. \end{cases} \tag{13}$$

The generator satisfies $q_{ij} \geq 0$ for $i \neq j$, $q_{ii} \leq 0$, and $\sum_j q_{ij} = 0$ for all i.

The transition probability matrix over a time interval t is

$$P(t) = [p_{ij}(t)] = e^{Qt} = \sum_{k=0}^{\infty} \frac{(Qt)^k}{k!}, \tag{14}$$

where

$$p_{ij}(t) = P(Y(t) = s_j \mid Y(0) = s_i) = [e^{Qt}]_{ij}. \tag{15}$$

This satisfies the Kolmogorov forward equation

$$\frac{d}{dt}P(t) = P(t)Q = QP(t), \qquad P(0) = I. \tag{16}$$

If the chain is irreducible and positive recurrent, the stationary distribution $\pi = [\pi_1, \dots, \pi_m]$ satisfies

$$\pi Q = 0, \qquad \sum_{i=1}^{m} \pi_i = 1, \qquad \pi_i \geq 0. \tag{17}$$

The expected holding time in state S_i before a jump occurs is

$$E[T_i] = \frac{1}{-q_{ii}} = \frac{1}{\lambda(1 - p_{ii})}. \tag{18}$$

Given observed transitions n_{ij} from state i to state j over T periods, the jump intensity is estimated as

$$\hat{\lambda} = \frac{\text{Number of observed jumps}}{T}, \tag{19}$$

and the transition probabilities are estimated by

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{k=1}^{m} n_{ik}}, \qquad \sum_{k=1}^{m} n_{ik} > 0. \tag{20}$$

The estimated generator is

$$\hat{Q} = \hat{\lambda}(\hat{P} - I). \tag{21}$$

These formulations define the model used to simulate paths, compute $P(t)$, and derive risk metrics such as value-at-risk and expected shortfall from the distribution of $Y(t)$.

3. Results and Discussion

The three-state CTMC analysis of BUA Cement and BZU Cement over a nine-month period reveals distinct regime-switching structures despite both equities operating in the NGX cement sector. States were defined using firm-specific price thresholds: BUA Low

< 90, Mid 90–140, High ≥ 140; and BZU Low < 44, Mid 44–46, High ≥ 46. The estimated generator matrices are

$$Q_{\text{BUA}} = \begin{pmatrix} -0.333 & 0.333 & 0 \\ 0.250 & -0.500 & 0.250 \\ 0 & 0 & 0 \end{pmatrix}, \qquad Q_{\text{BZU}} = \begin{pmatrix} -0.667 & 0.333 & 0.333 \\ 0.333 & -1.000 & 0.667 \\ 0.333 & 0.333 & -0.667 \end{pmatrix}. \quad (22)$$

Stationary distributions obtained from $\pi Q = 0$ are $\pi_{\text{BUA}} = (0.300, 0.400, 0.300)$ and $\pi_{\text{BZU}} = (0.333, 0.333, 0.333)$. Expected holding times $E[T_i] = 1/|q_{ii}|$ show that BUA persists for 3.00 months in Low and 2.00 months in Mid, while High is absorbing in the observed sample because $q_{33} = 0$. BZU exits all regimes faster, with 1.50, 1.00, and 1.50 months for Low, Mid, and High respectively. The total switching intensity is 0.833 for BUA and 2.000 for BZU, indicating that BZU switches regimes about 2.4 times faster.

Kolmogorov forecasts from the High regime, $P(3) = \exp(3Q)$, give $P_{\text{BUA},3j}(3) = (0\%, 0\%, 100\%)$ and $P_{\text{BZU},3j}(3) = (33.6\%, 32.2\%, 34.2\%)$. Thus, BUA exhibits trend persistence with an absorbing bull state and longer-lived bear/neutral regimes, while BZU exhibits anti-persistent, ergodic behaviour with nearly uniform time across regimes and a 33.6% probability of moving from High to Low within three months. The structural difference is that BUA's generator is not fully connected due to $q_{31} = q_{32} = 0$, whereas BZU is irreducible with positive off-diagonal transition rates.

Table 1: Summary of Continuous-Time Markov Chain metrics for BUA and BZU Cement.

Metric	BUA Cement	BZU Cement	Interpretation
State definition	Low: < 90 Mid: 90–140 High: ≥ 140	Low: < 44 Mid: 44–46 High: ≥ 46	Firm-specific price thresholds based on observed ranges.
Generator matrix Q	Q_{BUA}	Q_{BZU}	BUA has $q_{31} = q_{32} = 0$; BZU has all off-diagonal rates positive.
Stationary distribution π	$(0.300, 0.400, 0.300)$	$(0.333, 0.333, 0.333)$	BUA concentrates more in the Mid regime; BZU has equal long-run time in all regimes.
Expected holding time $E[T_i]$	Low: 3.00 months Mid: 2.00 months High: ∞	Low: 1.50 months Mid: 1.00 month High: 1.50 months	BUA stays longer in Low/Mid; BZU exits all regimes within 1.5 months.
Total switching intensity	0.833	2.000	BZU switches regimes about 2.4 times faster than BUA.

Metric	BUA Cement	BZU Cement	Interpretation
Three-month forecast from High	Low: 0.0% Mid: 0.0% High: 100.0%	Low: 33.6% Mid: 32.2% High: 34.2%	BUA remains High; BZU has substantial probability of returning to Low.
Regime type	Trend-persistent; absorbing High	Ergodic; mean-reverting	BUA is path-dependent; BZU shows rapid regime turnover.

Table 2: Regime transition probabilities for a one-month horizon.

Stock	From → To	Low	Mid	High
BUA	Low	0.717	0.283	0.000
	Mid	0.221	0.607	0.221
	High	0.000	0.000	1.000
BZU	Low	0.513	0.283	0.283
	Mid	0.283	0.368	0.368
	High	0.283	0.283	0.513

From the High regime, BUA has a 100% chance of remaining High after one month, whereas BZU has only a 51.3% chance. BZU also has a 28.3% chance of a direct High-to-Low transition within one month, while BUA has 0%. This quantifies BZU's higher regime instability, as shown in Table 2.

3.1. Graphical Analysis of Regime-Switching Metrics

This subsection presents the graphical interpretation of the CTMC results for BUA Cement and BZU Cement. The plots summarize the stationary regime structure, expected holding time, switching intensity, one-month transition probability matrices, and three-month forecasts from the High regime.

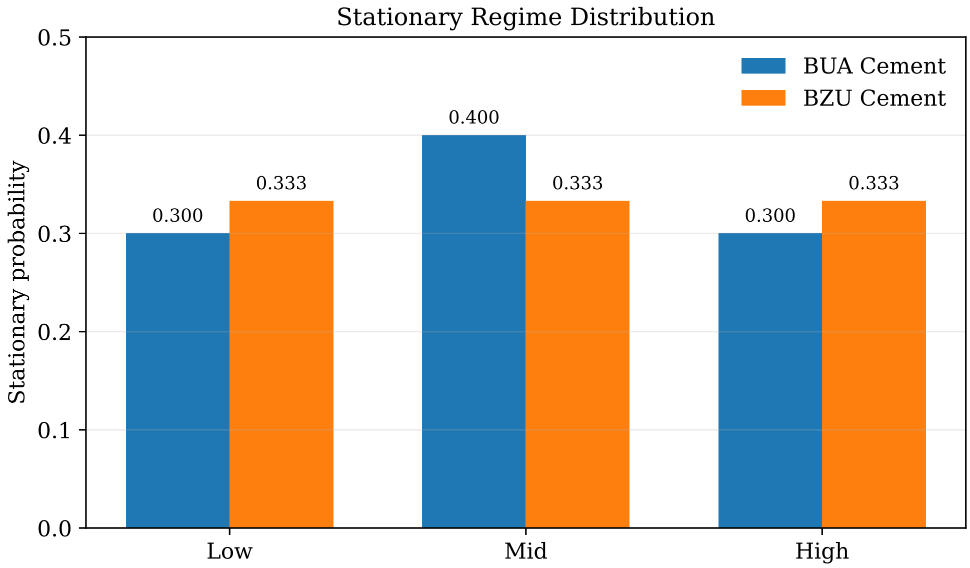


Figure 1: Stationary distribution of BUA and BZU Cement across Low, Mid, and High regimes.

Figure 1 shows that BUA Cement has a higher stationary concentration in the Mid regime, while BZU Cement is evenly distributed across Low, Mid, and High regimes. This suggests that BUA has stronger regime concentration, whereas BZU displays a more balanced long-run movement across all regimes.

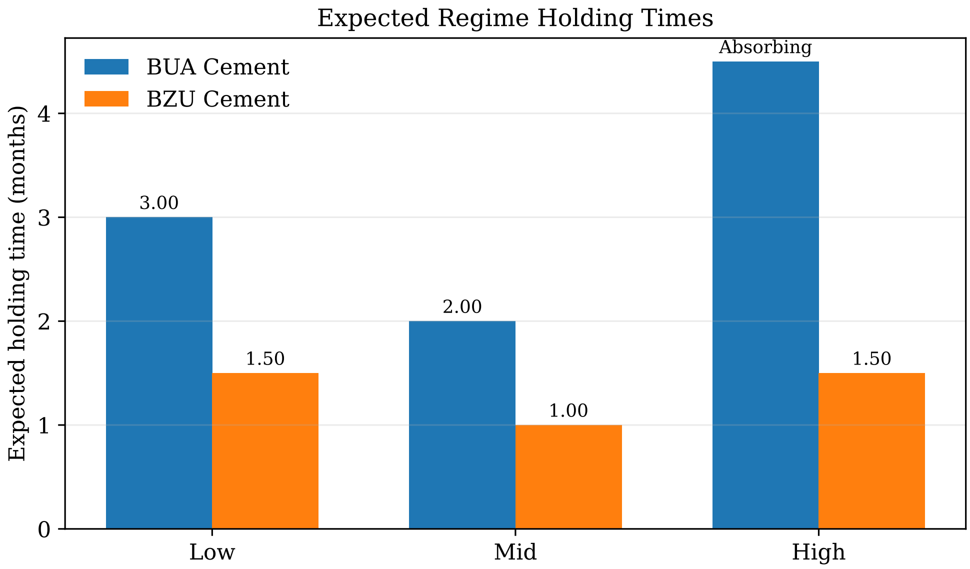


Figure 2: Expected holding times for BUA and BZU Cement. The High regime for BUA is absorbing in the sample.

Figure 2 indicates that BUA remains longer in the Low and Mid regimes than BZU. BUA has expected holding times of 3.00 months in Low and 2.00 months in Mid, while BZU exits the Low, Mid, and High regimes within 1.50, 1.00, and 1.50 months respectively. The absorbing High regime for BUA suggests strong upward trend persistence in the observed period.

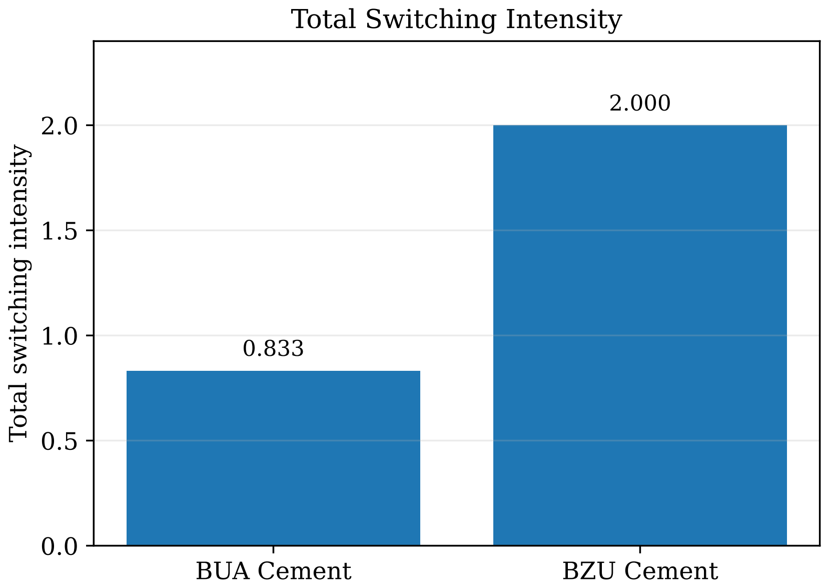
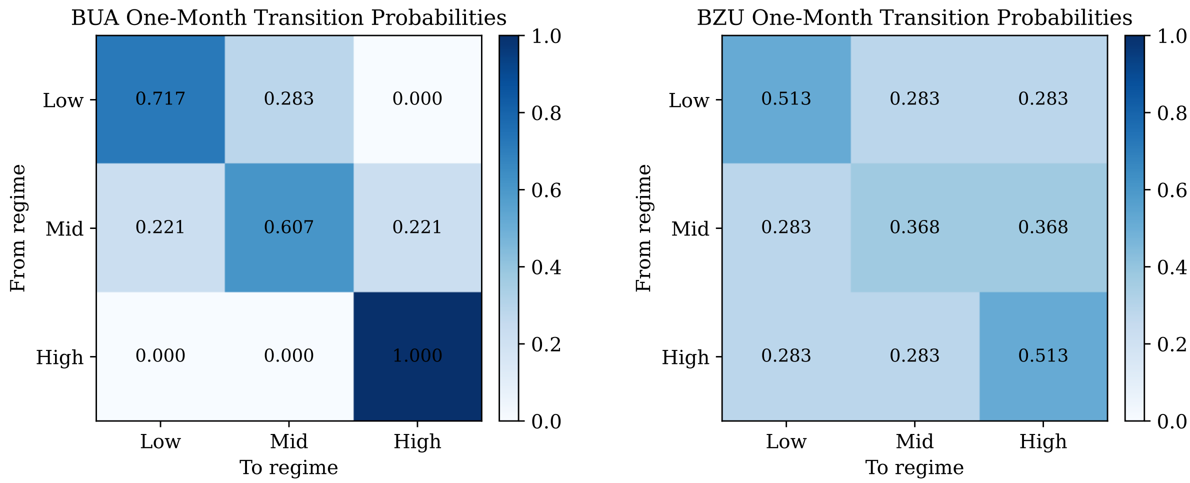


Figure 3: Total switching intensity for BUA and BZU Cement.

Figure 3 confirms that BZU Cement switches regimes faster than BUA Cement. The total switching intensity of BZU is 2.000 compared with 0.833 for BUA. This supports the result that BZU is more regime-volatile and is more exposed to frequent changes in price states.



(a) BUA Cement

(b) BZU Cement

Figure 4: One-month transition probability heatmaps for BUA and BZU Cement.

Figure 4 presents the one-month transition probabilities. BUA has a 100% probability of remaining in the High regime once it enters that state, confirming the absorbing High state. In contrast, BZU has only a 51.3% probability of remaining High and a 28.3% probability of falling directly from High to Low. This shows that BZU has higher short-term downside regime risk.

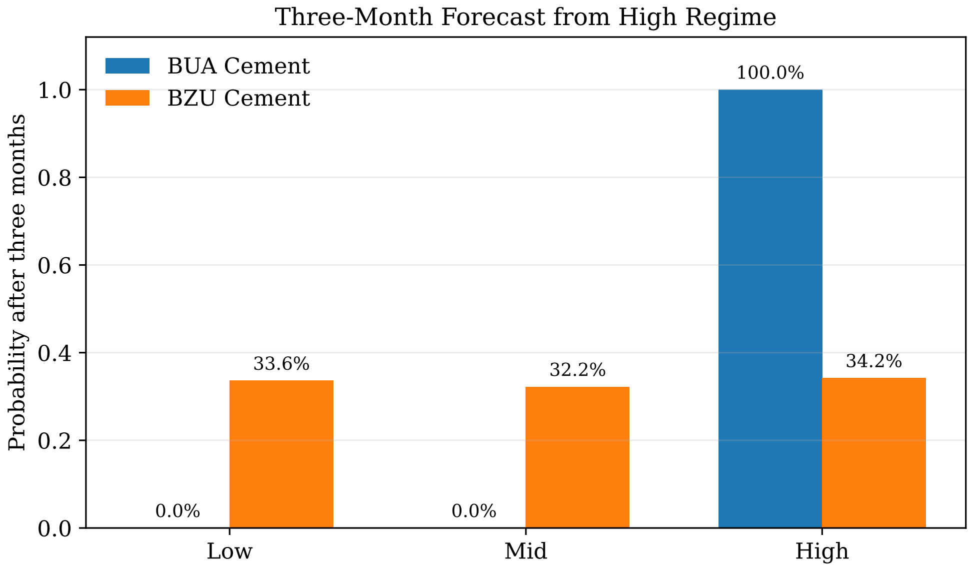


Figure 5: Three-month forecast from the High regime for BUA and BZU Cement.

Figure 5 shows that after three months from the High regime, BUA remains in the High regime with probability 100%, while BZU spreads across Low, Mid, and High regimes with 33.6%, 32.2%, and 34.2% respectively. This confirms that BUA is trend-persistent, whereas BZU is mean-reverting and regime-unstable.

4. Conclusion

BUA and BZU Cement show opposite regime dynamics. BUA has an absorbing High state, longer stays in Low and Mid regimes, lower switching intensity, and a long-run tilt toward Mid/High regimes. BZU is fully ergodic with fast switching, short regime durations, and a uniform long-run split across all states, giving it higher crash risk from the High regime. The one-month transition matrix for BUA has an absorbing High state with zero exit risk, while BZU's matrix reveals frequent switching and direct High-to-Low crash risk. These findings confirm that CTMC analysis can isolate regime-switching risk as a property distinct from ordinary price volatility. The main limitation is that BUA's absorbing High state is based on the observed sample period; longer data are needed to test whether this absorption is a stable structural feature.

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Conflict of Interest

The authors declare no conflict of interest.

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